Supplementary Material

Nanomechanical vibrational response from electrical mixing measurements

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I: TWO-SOURCE MIXING METHOD

We consider a double-clamped mechanical resonator that is capacitively coupled to an immobile gate electrode. The two-source mixing method requires that the conductance through the resonator varies when sweeping the gate voltage. In what follows, we consider the regime of single-electron tunneling, but the same final result for the mixing current is obtained for any other regime. The vibrations are driven by applying an oscillating voltage $V_{\mathrm{g}}^{\mathrm{ac}} \cos \omega t$ on the gate electrode. When applying the voltage $V_{\mathrm{s}}^{\mathrm{ac}} \cos\left((\omega+\delta\omega)t + \varphi_{\mathrm{e}}\right)$ on the source electrode, the mixing current at frequency $\delta\omega$ arises in the Taylor expansion of the current in z and V_{g} [1]. The dependence of the current on these two quantities can be traced back to the tunnelling rate dependence on the electrostatic energy difference between the two relevant charge states of the dot. For vanishing bias voltage the electrostatic energy reads $E_E(Q) = (Q + C_g(z)V_g)^2/2C_\Sigma(z)$ with C_g and C_Σ the gate and total capacitances and Q the charge on the dot. This gives for the relevant energy difference $\Delta E = E_E(Q-e) - E_E(Q) = e(e/2 - Q - C_g(z)V_g)/C_\Sigma(z)$. The current is then a function of $\Delta E(z, V_{\mathrm{g}})$. Expanding the current expression for $eV_{\mathrm{s}} \ll k_B T$, small displacement z [given by Eq. 1 of the main text], and $V_{\mathrm{g}}^{\mathrm{ac}}$ one obtains:

$$I = \frac{\partial G}{\partial V_{\mathrm{g}}} V_{\mathrm{s}}^{\mathrm{ac}} \cos\left((\omega+\delta\omega)t+\varphi_{\mathrm{e}}\right) \times \left[V_{\mathrm{g}}^{\mathrm{ac}} \cos\omega t + V_{\mathrm{g}}^{\mathrm{dc}} C'_{\mathrm{g}}/C_{\mathrm{g}}[X_{\mathrm{z}}\cos(\omega t) + Y_{\mathrm{z}}\sin(\omega t)]\right]. \tag{S1}$$

Here $\partial G/\partial V_{\mathrm{g}}$ is the transconductance of the nanotube device, $V_{\mathrm{g}}^{\mathrm{dc}}$ the static voltage applied to the gate and we assumed $Q \approx -C_{\mathrm{g}}V_{\mathrm{g}} \gg e$. Expanding the argument of the first cosine and averaging over a period $2\pi/\omega$ gives the mixing current $I^{\delta\omega}$ at frequency $\delta\omega$:

$$I^{\delta\omega} = \frac{1}{2}\frac{\partial G}{\partial V_{\mathrm{g}}} V_{\mathrm{s}}^{\mathrm{ac}} \left[\cos\left(\delta\omega t + \varphi_{\mathrm{e}}\right)\left(V_{\mathrm{g}}^{\mathrm{ac}} + \frac{V_{\mathrm{g}}^{\mathrm{dc}}C'_{\mathrm{g}}}{C_{\mathrm{g}}}X_{\mathrm{z}}\right) - \sin\left(\delta\omega t+\varphi_{\mathrm{e}}\right)\frac{V_{\mathrm{g}}^{\mathrm{ac}}C'_{\mathrm{g}}}{C_{\mathrm{g}}}Y_{\mathrm{z}}\right]. \tag{S2}$$

This leads to the mixing current quadratures X_{I} and Y_{I}, which depend on the quadratures X_{z} and Y_{z} of the displacement as

$$X_{\mathrm{I}} = \alpha\left[(X_{\mathrm{z}} + C_{\mathrm{g}}V_{\mathrm{g}}^{\mathrm{ac}}/C'_{\mathrm{g}}V_{\mathrm{g}}^{\mathrm{dc}})\cos\varphi_{\mathrm{e}} - Y_{\mathrm{z}}\sin\varphi_{\mathrm{e}}\right], \tag{S3}$$

$$Y_{\mathrm{I}} = \alpha\left[-(X_{\mathrm{z}} + C_{\mathrm{g}}V_{\mathrm{g}}^{\mathrm{ac}}/C'_{\mathrm{g}}V_{\mathrm{g}}^{\mathrm{dc}})\sin\varphi_{\mathrm{e}} + Y_{\mathrm{z}}\cos\varphi_{\mathrm{e}}\right], \tag{S4}$$

with $\alpha = (\partial G/\partial V_{\mathrm{g}})V_{\mathrm{s}}^{\mathrm{ac}}V_{\mathrm{g}}^{\mathrm{dc}}C'_{\mathrm{g}}/2C_{\mathrm{g}}$. The expression of the mixing current in Eq. S2 and its quadratures in Eqs. S3 and S4 are the same for other types of conductors, such as the electronic Farby-Pérot interferometer or the the field-effect transistor [1]. In the next section, we show that the capacitive force in the single-electron tunneling regime is different from that in other regimes.

II: DRIVING FORCE IN THE SINGLE-ELECTRON TUNNELING REGIME

We discuss here the oscillating force acting on a mechanical resonator hosting a dot that behaves as a single-electron transistor in the limit typically realized in experiments with a slow oscillator $\Gamma \gg \omega_m$, where Γ is the typical incoherent tunneling rate ($k_BT \gg \hbar\Gamma$). When the gate voltage is modulated, the charge on the dot changes, leading to an additional oscillating force acting on the oscillator. This is the reason why the constant β in Eq. 6 in the main text for the capacitive force can deviate from one. The total capacitive force between the resonator and the the gate electrode can be written as

$$F = -\frac{\partial}{\partial z}\frac{Q_g^2}{2C_g(z)} = \frac{Q_g^2 C'_g}{2C_g^2} \tag{S5}$$

where Q_g is the charge on the gate electrode (we assume that the capacitances to the source or drain are not modified by the displacement of the resonator). In the sequential tunnelling regime the charge on the dot is always an integer multiple of the elementary charge $Q = -e(n_0+n)$, with n_0 and n integers, and only n varies between 0 and 1. From electrostatics the gate charge is then:

$$Q_g = C_gV_g - \frac{C_g}{C_\Sigma}(V_sC_s + C_dV_d + C_gV_g + Q). \tag{S6}$$

where we introduced the source and drain voltages (V_s,V_d) and capacitances (C_s, C_d) with $C_\Sigma = C_g + C_s + C_d$. Since the number of electrons fluctuates of one unit during transport, there are actually two forces acting on the dot, one for each value of Q. Using the separation of time scales we can assume that the oscillator cannot respond to the fast electron fluctuations, and thus it feels an average force given by the average value of Q. When $\delta V_g(t) = V_g^{\rm ac}\cos(\omega t)$ is applied to the gate electrode, we can write that the resulting variation of the charge on the gate electrode reads:

$$\delta Q_g = C_g \delta V_{\rm g}\left[1 - \frac{C_g}{C_\Sigma}\right] + \frac{C_g}{C_\Sigma}\delta\langle Q\rangle. \qquad \text{(S7)}$$

We can neglect the higher orders in the z-dependence of the capacitance when computing the force in Eq. S5, since this gives rise only to a renormalization of the resonance frequency. The variation of Q is controlled by the master equation for the charge. Assuming that only two charge states are possible, one has $\langle Q\rangle = -n_0 e - ef$ with f the Fermi function $f = (e^{\varepsilon/k_B T} + 1)^{-1}$ where the ε dependence on the gate voltage is $\delta\varepsilon = -eC_g\delta V_g/C_\Sigma$. We obtain then

$$\delta\langle Q\rangle = -\frac{e^2}{k_B T}\frac{C_g}{C_\Sigma}\delta V_g f(1-f). \qquad \text{(S8)}$$

Note that the factor $(e^2/C_\Sigma)/k_B T \gg 1$ in the Coulomb blockade regime. This term $\delta\langle Q\rangle$ is largest for gate voltages at which the peak conductance is highest and where $f = 1/2$. Inserting Eq. S8 into Eq. S7 and Eq. S5 one obtains Eq. 5 of the main text.

For completeness, it can be useful to recall the derivation of the coupling constant between the mechanical and electronic degrees of freedom. This is the variation of the force acting on the oscillator when an electron on the dot is added or removed.

$$F_0 = F_g(Q) - F_g(Q-e) = \frac{C_g' e^2}{C_g C_\Sigma}(Q_g/e - 1). \qquad \text{(S9)}$$

For $|Q_g| \gg e$ and $V_s \approx V_d \approx 0$ one finds

$$F_0 = \frac{C_g' V_g e}{C_\Sigma}. \qquad \text{(S10)}$$

[1] V. Sazonova, Y. Yaish, H. Üstünel, D. Roundy, T. A. Arias, and P. L. McEuen, Nature **431**, 284 (2004).