Supplementary Information
Ultrasensitive displacement noise measurement of carbon nanotube mechanical resonators

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I. TRANSDUCTION OF DISPLACEMENT INTO CURRENT

We summarize the important relations for the transduction of displacement into current [1]. We use the fact that the mechanical eigenmode is polarized in the direction perpendicular to the surface of the gate electrode, as discussed in the main text. The current δI at the frequency close to the difference between the mode eigenfrequency and the frequency of the source-drain voltage is

$$\delta I = \beta \delta z, \tag{1}$$

$$\beta = \frac{1}{2}\frac{dG}{dV_{\mathrm{G}}} V_{\mathrm{G}}^{\mathrm{dc}} V_{\mathrm{sd}}^{\mathrm{ac}} \frac{C'_{\mathrm{G}}}{C_{\mathrm{G}}}. \tag{2}$$

Here, δz is the displacement of the nanotube, dG/dV_{G} is the transcondutance, $V_{\mathrm{G}}^{\mathrm{dc}}$ is the static gate voltage, $V_{\mathrm{sd}}^{\mathrm{ac}}$ is the amplitude of the oscillating source-drain voltage, C_{G} is the capacitance between the nanotube and the gate electrode, and C'_{G} is the derivative of C_{G} with respect to z. We measure $dG/dV_{\mathrm{G}} = 1.4 \times 10^{-3}$ S/V at the gate voltage discussed in the main text.

We estimate the capacitance C_{G} from the separation $\Delta V_{\mathrm{G}}^{\mathrm{dc}} = 16.8 \pm 0.6$ mV between two conductance peaks in the Coulomb blockade regime at large positive $V_{\mathrm{G}}^{\mathrm{dc}}$ values (Fig. 1). We obtain $C_{\mathrm{G}} = e/\Delta V_{\mathrm{G}}^{\mathrm{dc}} = 9.5 \pm 0.3 \times 10^{-18}$ F. We quantify C'_{G} using the relation

$$C'_{\mathrm{G}} = \frac{C_{\mathrm{G}}}{d \ln(2d/r)} = (1.11 \pm 0.17) \times 10^{-11} \mathrm{F/m}, \tag{3}$$

with $d = 150 \pm 10$ nm for the separation between the nanotube and the gate electrode and $r = 1 \pm 0.3$ nm for the radius of the nanotube.

The spectral density S_{zz} of the displacement noise in the main text is obtained from the measured spectral density of the current noise using Eqs. 1 and 2. The displacement sensitivity $S_{\mathrm{zz}}^{\mathrm{imp}}$ in the main text is estimated from the current noise floor $S_{\mathrm{II}}^{\mathrm{imp}}$ in the measured

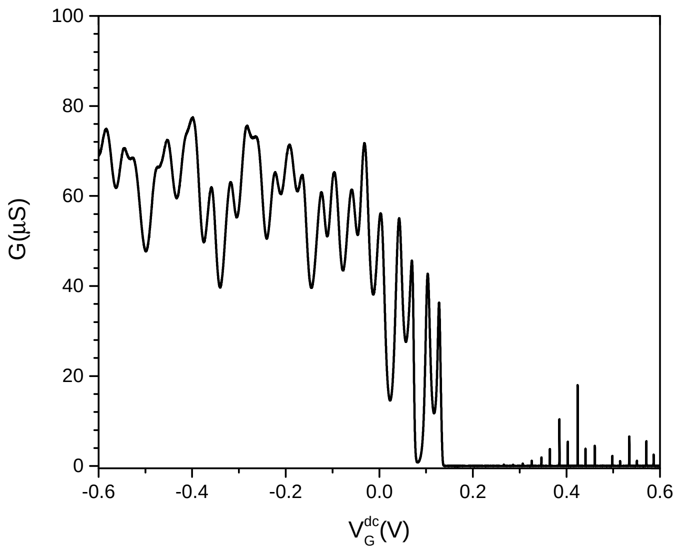

FIG. 1. Conductance of the nanotube as a function of gate voltage measured at the base temperature of the cryostat.

spectrum using

$$S_{\rm zz}^{\rm imp} = \frac{1}{\beta^2} S_{\rm II}^{\rm imp}. \tag{4}$$

II. CHARACTERISTICS OF THE NANOTUBE DISCUSSED IN THE MAIN TEXT

We estimate the effective mass $m = 8.6 \pm 3.6$ ag from the measurement of the variance of the displacement $\langle \delta z^2 \rangle$ as a function of temperature T in Fig. 2d of the main text. For this, we compare the measured slope of $\langle \delta z^2 \rangle$ as a function of T to the slope expected from the equipartition theorem, which reads

$$m\omega_0^2 \langle \delta z^2 \rangle = k_b T, \tag{5}$$

with $\omega_0/2\pi$ the resonance frequency of the eigenmode. This mass is consistent with the mass of a ~ 1.3 μm long nanotube. We obtain the same mass through a second measurement, where we compare the amplitude of the driving vibration amplitude to the amplitude of the thermal vibrations, as discussed in the supplementary Information of Ref. [1].

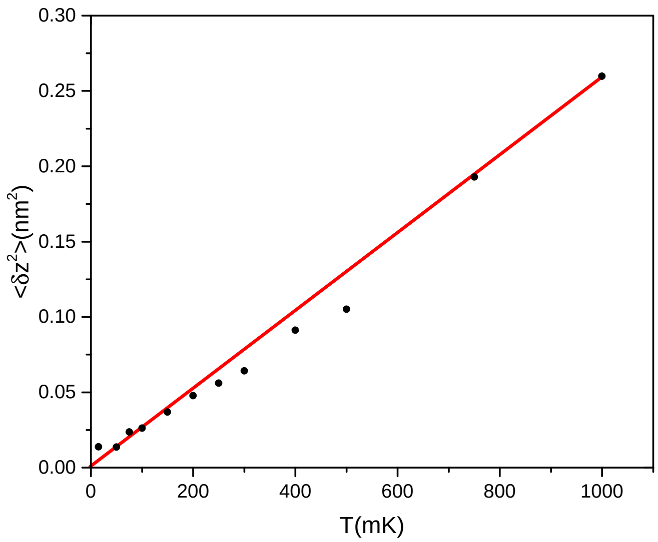

FIG. 2. Variance of the displacement of a second nanotube resonator as a function of the temperature of the cryostat.

Because the thermal resonance is described by a Lorentzian line shape in Fig. 2c of the main text, the force sensitivity is quantified from the total displacement noise at resonance frequency using

$$\sqrt{S_{\mathrm{FF}}} = \sqrt{S_{\mathrm{zz}}(\omega_0)}\frac{m\omega_0^2}{Q} = (4.3 \pm 2.9) \times 10^{-21}\mathrm{N}/\sqrt{\mathrm{Hz}}, \tag{6}$$

where Q is the quality factor measured from the linewidth in the spectrum. The error bar in the estimation of the force sensitivity originates essentially from the uncertainty in d and r.

The electrical characteristics of the nanotube in Fig. 1 is typical of ultraclean nanotubes [1]. For large positive $V_{\mathrm{G}}^{\mathrm{dc}}$ values, $p-n$ junctions are formed near the metal electrodes, forming a Coulomb blockaded region along the suspended nanotube. For negative $V_{\mathrm{G}}^{\mathrm{dc}}$, the nanotube is p-doped along the whole tube, resulting in a larger conductance.

III. THERMALIZATION OF A SECOND NANOTUBE RESONATOR

Figure 2 shows the variance of the displacement as a function of temperature. We observe a linear dependence of $\langle\delta z^2\rangle$ as a function of T at high temperature. The variance $\langle\delta z^2\rangle$ at the base temperature of the cryostat corresponds to a temperature of ~ 50 mK.

[1] J. Moser, A. Eichler, J. Guttinger, M. I. Dykman, and A. Bachtold, Nature Nanotechnology **9**, 1007 (2014).