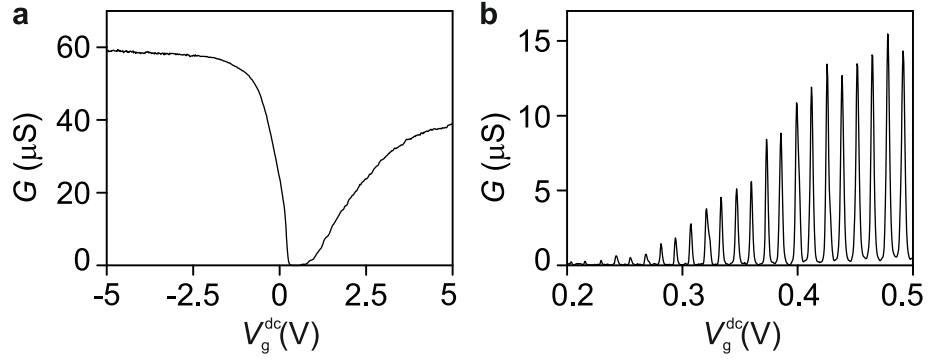
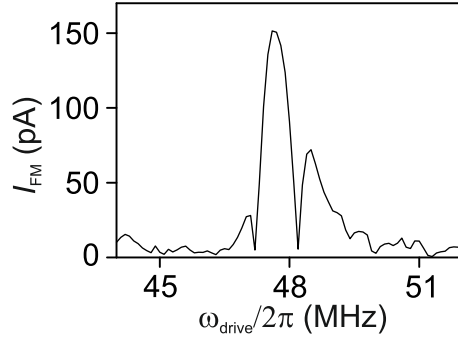


**Supplementary Figure S1. Measurements for a second nanotube resonator.** This device has the same geometrical layout as the one discussed in the main text. All measurements are performed at 65 K. **a**, Current as a function of angular driving frequency  $\omega_{\text{drive}}$  and static gate voltage  $V_g^{\text{dc}}$  measured with the FM technique.  $V_{\text{FM}} = 1.1$  mV. Color bar: 0 (white) to 20 pA (dark red). **b**, Low-frequency current  $I_{\text{LF}}$  versus  $\omega_{\text{drive}}$  with  $V_g^{\text{ac}} = 0.53$  mV, static bias voltage  $V_{\text{sd}}^{\text{dc}} = 10$  mV, and  $V_g^{\text{dc}} = 2.1$  V. **c**,  $X$  quadrature ( $I_{\text{vibra}}^X$ ) and **d**,  $Y$  quadrature ( $I_{\text{vibra}}^Y$ ) of the current measured with the 2-source mixing technique with  $V_g^{\text{ac}} = 1.8$  mV,  $V_{\text{sd}}^{\text{ac}} = 0.6$  mV, and  $V_g^{\text{dc}} = 2.1$  V. A small shift of the resonance frequency occurred relative to the measurement in **b**.



**Supplementary Figure S2. Nanotube conductance as a function of static gate voltage.**

**a**, Nanotube conductance at 65 K as a function of the constant gate voltage  $V_g^{\text{dc}}$  applied to the gate electrode. **b**, Nanotube conductance at 650 mK. The device is in the Coulomb blockade regime. The spacing between consecutive conductance peaks is  $\Delta V_g = eC_g$ , where  $e$  is the electron charge and  $C_g$  the capacitance of the nanotube with respect to the gate electrode.



**Supplementary Figure S3. Mechanical resonance at room temperature.** Mechanical vibration of the same nanotube device as in the main text measured at 250 K with the FM technique. The resonance width  $\omega_0/2\pi Q$  can be conveniently read out from the separation between the two minima that are flanking the main peak [4], with  $\omega_0$  the angular resonance frequency and  $Q$  the quality factor. We obtain  $Q \sim 50$ . We used a static gate voltage  $V_g^{\text{dc}} = -0.6$  V and  $V_{\text{FM}} = 5$  mV.

## SUPPLEMENTARY NOTE 1: ADDITIONAL DEVICE

In this section, we present data from a second nanotube device. We verify with atomic force microscopy that the trench width and depth are the same as for the device presented in the main text (1.8  $\mu\text{m}$  and 350 nm, respectively). The roughness of the metal electrodes do not allow a measurement of the nanotube diameter. We therefore use the same estimates for the capacitance and the mass as for the first device ( $C_g = 12 \text{ aF}$ ,  $\partial_z C_g = 5.6 \text{ pF/m}$ ,  $\partial_z^2 C_g = 21 \mu\text{F/m}^2$ , and  $m \simeq 4 \text{ ag}$ ).

In the studied frequency range, we detect two mechanical resonances that change with an applied gate voltage (supplementary Fig. S1a). In the following, we concentrate on the second visible mode with a static gate voltage  $V_g^{\text{dc}} = 2.1 \text{ V}$ . We measure the low-frequency current  $I_{\text{LF}}$  and find a peak at the resonance frequency (supplementary Fig. S1b). In order to determine the vibration amplitude  $z_{\text{vibra}}$ , we also measure  $I_{\text{vibra}}^X$  and  $I_{\text{vibra}}^Y$  with the 2-source mixing technique (supplementary Fig. S1c and d). We obtain a vibrational amplitude  $z_{\text{vibra}} = 8.9 \text{ nm}$  for a driving voltage of 1.8 mV. Unfortunately, the signal-to-noise ratio of the 2-source mixing technique was not sufficient to measure  $z_{\text{vibra}}$  directly for the lower driving voltage used to measure  $I_{\text{LF}}$ . In order to compare the two sets of data, we assume  $z_{\text{vibra}} \propto V_g^{\text{ac}}$ , thus obtaining the scaled value  $z_{\text{vibra}} = 2.6 \text{ nm}$  for  $V_g^{\text{ac}} = 0.53 \text{ mV}$ .

We perform the same analysis as for the main device to identify the origin of  $I_{\text{LF}}$ . We estimate the currents due to the nonlinearities in the capacitance and in the electrical conductance, finding  $I_{\text{LF}}^{\text{capa}} = 4.8 \text{ pA}$  and  $I_{\text{LF}}^{\text{cond}} = 0.21 \text{ pA}$ . Both values are far below the measured  $I_{\text{LF}}^{\text{max}} = 374 \text{ pA}$ . In addition to  $C_g$ ,  $\partial_z C_g$ , and  $\partial_z^2 C_g$  mentioned above, we use here  $V_{\text{sd}}^{\text{dc}} = 10 \text{ mV}$ ,  $V_{\text{sd}}^{\text{ac}} = 0.6 \text{ mV}$ , and  $\partial_{V_g}^2 G = 19 \mu\text{m/V}^2$ . For the analysis, we use an offset for  $V_g^{\text{dc}}$  such that  $V_g^{\text{dc}} = 0$  when  $\omega_0$  is lowest. As for the vibration amplitude, we use a scaled value  $I_{\text{vibra}}^Y = 30 \text{ pA}$  to account for the difference in the driving voltages.

We now consider symmetry breaking of the vibrations as the origin of the peak in  $I_{\text{LF}}$ . Using Eq. (19), we calculate  $\beta = 1.8 \cdot 10^{25} \text{ m}^{-1}\text{s}^{-2}$ . This value can be compared to the one we obtain from the dependence of  $\omega_0$  on  $V_g^{\text{dc}}$ . From Eq. (S13), we get  $\beta = 5.2 \cdot 10^{24} \text{ m}^{-1}\text{s}^{-2}$ . For this device, we have not studied the dependence of  $\omega_0$  on  $V_g^{\text{ac}}$ .

The fluctuation-induced spectral broadening  $\overline{\delta\omega}$ , expected from the two values of  $\beta$ , leads to a quality factor at room temperature that lies between 7.3 and 88 in the absence of cubic nonlinearity.

While we have not measured the quality factor at room temperature with this device, we note that such values are common for nanotube resonators.

## SUPPLEMENTARY DISCUSSION

### General expression for the low-frequency conductance

We consider the conductance of a suspended nanotube [42, 43] in the presence of a gate voltage that has a large DC component and a small AC component at a frequency close to the frequency of the eigenvibrations of the nanotube.

We assume that (i) the nanotube conductance is a function only of the total charge  $q$  of the nanotube, (ii) the charge distribution along the nanotube is independent of the gate voltage  $V_g$ , and (iii) the system is in the adiabatic limit, i.e. the vibration dynamics is much slower than the electron dynamics. Then  $q$  is related to  $V_g$  by the capacitance  $C_g$ . We consider the effect on conductance of the bending mode of the nanotube which is polarized in the direction  $z$  perpendicular to the gate.

Based on the above assumptions we write the conductance as

$$G(q(t)) \simeq G(q_0) + \partial_q G \delta q(t) + \frac{1}{2} \partial_q^2 G [\delta q(t)]^2 + \dots \quad (\text{S1})$$

For the temperatures used in our experiments, where the Coulomb blockade effect is insignificant, the term  $\propto \delta q^2$  is comparatively small. Its effect is discussed in Methods (Estimation of  $I_{\text{LF}}$  due to conductance nonlinearity), and also below Eq. (S10).

The charge increment  $\delta q(t)$  is a function of the time-dependent (AC) increment of the gate voltage  $\delta V_g(t)$  and the (AC) vibrational displacement  $\delta z(t)$ , which is the displacement of the nanotube at the antinode of the vibrational mode with the largest amplitude, for a given mode. For small  $|\delta V_g|$  and  $|\delta z|$

$$\delta q(t) \simeq \partial_{V_g} q \delta V_g(t) + \partial_z q \delta z(t) + \frac{1}{2} \partial_{V_g}^2 q \delta V_g(t)^2 + \partial_z \partial_{V_g} q \delta z(t) \delta V_g(t) + \frac{1}{2} \partial_z^2 q \delta z(t)^2. \quad (\text{S2})$$

The coefficients in this expression have a simple form in the case where the charge  $q$  is related to the gate voltage by the gate capacitance  $C_g$ , which itself depends on the displacement of the nanotube. We then have

$$\partial_{V_g} q = C_g, \quad \partial_z q = V_g^{\text{dc}} \partial_z C_g, \quad \partial_z \partial_{V_g} q = \partial_z C_g, \quad \partial_z^2 q = V_g^{\text{dc}} \partial_z^2 C_g, \quad (\text{S3})$$

whereas we can set  $\partial_{V_g}^2 q = 0$ , assuming the capacitor to be linear. In Eq. (S3)  $V_g^{\text{dc}}$  is the DC gate voltage, which is assumed to be large compared to  $\delta V_g(t)$ .

We will consider an AC modulation  $\delta V_g(t)$  at frequency  $\omega_{\text{drive}}$  close to the eigenfrequency of the nanotube  $\omega_0$ . If this modulation is not too weak, the major contribution to the AC displacement  $\delta z$  is the one induced by this resonant modulation whereas the thermal displacement can be disregarded.

The quantity of immediate interest to us is the quasi-static change of the conductance in response to  $\delta V_g$ . As explained in the main text, to detect this change we consider a periodic signal with slowly modulated amplitude,

$$\delta V_g(t) = V_g^{\text{ac}}(t) \cos \omega_{\text{drive}} t, \quad V_g^{\text{ac}}(t) = V_0(1 - \cos(\omega_{\text{AM}} t)), \quad \omega_{\text{AM}} \ll \omega_{\text{drive}}. \quad (\text{S4})$$

There are several contributions to the low-frequency response of the conductance to the modulation (S4). To study them we first estimate the response of the resonator to the modulation assuming that the resonator dynamics is linear,  $\delta z = \delta z^{\text{lin}}$ . The linearized equation of motion in the simplest case of viscous friction reads

$$\delta \ddot{z}^{\text{lin}} + 2\Gamma \delta \dot{z}^{\text{lin}} + \omega_0^2 \delta z^{\text{lin}} = \frac{F_d(t) \cos(\omega_{\text{drive}} t)}{m}, \quad F_d(t) = \partial_z C_g V_g^{\text{dc}} V_g^{\text{ac}}(t) \quad (\text{S5})$$

where  $m$  is the mass of the nanotube,  $2\Gamma = \omega_0/Q$  is the decay rate of the oscillator with quality factor  $Q$ , and  $F_d(t)$  is the AC driving force amplitude. In what follows we assume that the frequency of the amplitude modulation,  $\omega_{\text{AM}}$ , is small compared to the decay rate  $\Gamma$ , so that the induced vibrations adiabatically follow  $V_g^{\text{ac}}(t)$ . Then

$$\delta z^{\text{lin}}(t) = A^{\text{lin}}(\omega_{\text{drive}}, t) \cos(\omega_{\text{drive}} t - \phi), \quad A^{\text{lin}}(\omega_{\text{drive}}, t) = \frac{F_d(t)/m}{\sqrt{(\omega_0^2 - \omega_{\text{drive}}^2)^2 + 4\Gamma^2 \omega_{\text{drive}}^2}},$$

$$\phi = \arctan \left( \frac{2\Gamma \omega_{\text{drive}}}{\omega_0^2 - \omega_{\text{drive}}^2} \right). \quad (\text{S6})$$

From Eqs.(S4), and (S6) it follows that all terms that have a quadratic dependence on  $\delta V_g$  and  $\delta z$  in Eq. (S2) have a slowly varying part, which oscillates with period  $2\pi/\omega_{\text{AM}}$ . If the distance between the gate electrode and the nanotube is  $h$  (it is of the order of the depth of the trench,  $\sim 350$  nm), then  $\partial_z C_g \sim C_g/h$ . Then in the linear approximation the amplitude on resonance is  $A_{\text{res}} = F_d/2m\Gamma\omega_0 \sim C_g V_g^{\text{ac}} V_g^{\text{dc}}/m\Gamma\omega_0 h$ . The slowly varying parts of the last two terms in Eq. (S2) are

$$\sim C_g V_g^{\text{ac}} A^{\text{lin}}/h, \quad C_g V_g^{\text{dc}} (A^{\text{lin}})^2/h^2. \quad (\text{S7})$$

A simple estimate shows that, for the device parameters, the second term is much larger than the first for resonant driving, which means that the term  $\propto \delta V_g \delta z$  in Eq. (S2) should be disregarded in the analysis of the low-frequency conductance.

## Nonlinear response of the vibrational mode

In the linear approximation [Eq. (S6)], the term  $\propto \delta z(t)$  in the expression for the charge and thus the conduction modulation [Eq. (S2)] are oscillating at high frequencies  $\omega_{\text{drive}}, \omega_{\text{drive}} \pm \omega_{\text{AM}}$ . However, the vibrations of the nanotube are nonlinear, and this leads to the onset of slowly varying terms in the displacement  $\delta z(t)$ . To find these terms we write the part of the capacitive energy and the internal energy of the mode that is nonlinear in  $\delta V_g$  and  $\delta z$ ,

$$H_c^{\text{nl}} = -\frac{1}{2}\partial_z C_g \delta V_g^2 \delta z - \frac{1}{2}\partial_z^2 C_g V_g^{\text{dc}} \delta V_g \delta z^2 + \frac{1}{3}m\beta\delta z^3 + \frac{1}{4}m\gamma\delta z^4 + \dots \quad (\text{S8})$$

The first two terms in this expression describe the nonlinear capacitive energy, whereas the last two terms refer to the nonlinear part of the vibrational energy. We emphasize that the term which is cubic in  $\delta z$  is present only because the mode lacks inversion symmetry: this term is the indication of symmetry breaking (it corresponds to a force that is quadratic in  $\delta z$ ). Such symmetry breaking may result from the gate voltage which bends the nanotube. Therefore we expect that  $\beta$  depends on  $V_g^{\text{dc}}$ . On the other hand, the term  $\propto \gamma$  is the familiar Duffing nonlinearity, which has been known to play an important role in the vibrational dynamics of nanotubes [42, 43].

We emphasize again that  $\delta z$  refers to the maximal displacement for the considered mode in the  $z$ -direction, i.e., toward the gate electrode. More generally, for bending modes, one should think of the displacement  $\delta \mathbf{r}$  as a function of length  $l$  along the nanotube ( $\delta \mathbf{r}$  is locally transverse to  $dl$ ). Then, for example, the term that leads to  $(m/3)\beta\delta z^3$  in Eq. (S8) would be written as a triple integral over the length

$$H_{\text{sym-brk}} = \frac{1}{3}m\tilde{\beta} \int dl_1 dl_2 dl_3 f_{ijk}(l_1, l_2, l_3) \delta r_i \delta r_j \delta r_k. \quad (\text{S9})$$

Function  $f$  here is nonzero only for a nanotube with broken symmetry, i.e., where the energy changes if one replaces  $\delta \mathbf{r} \rightarrow -\delta \mathbf{r}$ . The term  $\propto \delta z^3$  in Eq. (S8) is obtained if one substitutes  $\delta \mathbf{r}(l)$  with the solution of the harmonic problem, for the considered mode. In general, in nanotubes with broken symmetry, the coupling between different modes leads to an energy that is cubic in the displacements of the modes.

A simple calculation shows that, to leading order, the first 3 terms in Eq. (S8) give the slowly varying terms in  $\delta z(t)$  of the form

$$\delta z_{\text{slow}}(t) \approx \partial_z C_g \frac{V_g^{\text{ac}}(t)^2}{4m\omega_0^2} + \partial_z^2 C_g V_g^{\text{dc}} \frac{V_g^{\text{ac}}(t) A^{\text{lin}}(\omega_{\text{drive}}, t)}{2m\omega_0^2} \cos \phi - \beta \frac{A^{\text{lin}}(\omega_{\text{drive}}, t)^2}{2\omega_0^2} \quad (\text{S10})$$

Here, the first term is very much smaller than the second term for typical device parameters; the ratio of these terms is of the same order of magnitude as the ratio of the terms in Eq. (S7). We

note, however, that on exact resonance,  $\omega_{\text{drive}} = \omega_0$ , we have  $\cos \phi = 0$ . Therefore either  $|\delta z_{\text{slow}}|$  displays an extremely narrow and extremely deep dip as a function of  $\omega_{\text{drive}}$ , which is expected for  $\beta \rightarrow 0$ , or the dominating term in Eq. (S10) is the last term, which comes from the broken inversion symmetry.

It is necessary also to look at the ratio of the contributions to the conductance modulation of the second term in  $\delta z_{\text{slow}}$  in Eq. (S10) and the term  $\partial_z^2 q \delta z^2$  in Eq. (S2). One can easily see that this ratio is  $\sim \Gamma/\omega_0 = (2Q)^{-1} \ll 1$ . Therefore the leading-order contribution to the scaled slowly varying conductance is

$$\delta G \approx \partial_q G V_g^{\text{dc}} \overline{\delta z(t)^2} \left( \frac{1}{2} \partial_z^2 C_g - \partial_z C_g \frac{\beta}{\omega_0^2} \right). \quad (\text{S11})$$

Here, bar means averaging over the period of fast oscillations  $2\pi/\omega_{\text{drive}}$ .

Experimentally, the easiest way to separate the two contributions to  $\delta G$  in Eq. (S11) is by estimating  $\partial_z C_g$ ,  $\partial_z^2 C_g$ , and  $\beta$  from independent measurements. In Methods, we demonstrate how this estimate is done for our device. The estimate indicates that the first term in the bracket in Eq. (S11) is too small to account for our measurements. We also estimate  $\beta$  by analyzing the shift of  $\omega_0$  as a function of  $z_{\text{vibra}}$  (the amplitude of  $\delta z(t)$ ). We find that this latter estimate is in good agreement with our measurement. Therefore, the major effect is coming from the symmetry breaking of the vibrations. In the main text and in the following, we refer to the slow motion  $\delta z_{\text{slow}}$  in terms of a (quasi-static) shift of the equilibrium position,  $\delta z_{\text{eq}}$ .

### The dependence of the vibration frequency on DC gate voltage

Here, we show that the quadratic nonlinearity of the restoring force leads to a shift of the nanoresonator frequency  $\Delta\omega_0$  due to a dc gate voltage. We start with equation of motion

$$m\ddot{z} + 2m\Gamma\dot{z} + m\omega_0^2 z = -m\beta z^2 - m\gamma z^3 + \frac{1}{2} \partial_z C_g (V_g^{\text{dc}})^2 + \frac{1}{2} \partial_z^2 C_g (V_g^{\text{dc}})^2 z, \quad (\text{S12})$$

where we assume that the displacement  $z$  is small. Taking  $z$  as the sum of a static contribution and an oscillating contribution, we get in first order in  $(V_g^{\text{dc}})^2$

$$\Delta\omega_0 = \frac{1}{2m\omega_0} \left[ \partial_z C_g \frac{\beta}{\omega_0^2} - \frac{1}{2} \partial_z^2 C_g \right] (V_g^{\text{dc}})^2. \quad (\text{S13})$$

The first term in the bracket, which depends on the symmetry breaking strength  $\beta$ , leads to the increase of  $\omega_0$  with  $V_g^{\text{dc}}$  ( $\beta$  is usually positive). This is the behavior observed for a large majority of nanotube resonators. By contrast, the second term leads to the decrease of  $\omega_0$  with  $V_g^{\text{dc}}$ . This is observed occasionally and is attributed to nanotubes with a large built-in tension.



Interestingly, the bracket of Eq. (S13) is the same as that of Eq. (S11) where the two terms correspond to the low-frequency currents induced by symmetry breaking and by the nonlinear capacitive coupling, respectively. In the resonator discussed in the main text,  $\omega_0$  increases with  $V_g^{\text{dc}}$ . This further supports our finding that the peak in  $I_{\text{LF}}$  is attributed to symmetry breaking.

We obtain from Fig. 4b in the main text that the prefactor  $a$  in the relation  $\Delta\omega_0 = a(V_g^{\text{dc}})^2$  ranges from  $4 \cdot 10^7$  to  $8 \cdot 10^7$ ; we offset  $V_g^{\text{dc}}$  so that  $V_g^{\text{dc}} = 0$  when  $\omega_0$  is minimum. From the length of the nanotube, we estimate that the effective mass is  $\simeq 4$  ag. Neglecting the second term in Eq. (S13), we obtain that  $\beta \simeq 3 \pm 1 \cdot 10^{24} \text{ m}^{-1} \text{ s}^{-2}$ . This value is comparable to the values estimated in the main text using different methods.

### Fluctuation-induced spectral broadening

Thermally-induced spectral broadening can be understood from Eq. (S12) if one incorporates into the right-hand side of this equation a random force  $f_{\text{T}}(t)$  that describes thermal noise. This noise comes from the same coupling to a thermal reservoir that leads to the friction force  $\propto \Gamma \dot{z}$ . From the fluctuation-dissipation relation the noise is  $\delta$ -correlated, with  $\langle f_{\text{T}}(t) f_{\text{T}}(t') \rangle = 4m\Gamma k_{\text{B}}T \delta(t - t')$ .

To gain a qualitative insight into the broadening we assume that the resonator vibrates as  $z_{\text{vibra}} \cos(\omega t + \phi)$  with frequency  $\omega$  close to  $\omega_0$ . We now look at the overall displacement as  $z(t) = z_{\text{vibra}} \cos(\omega t + \phi) + \delta z(t)$  and linearize Eq. (S12) with respect to  $\delta z(t)$ . The left-hand-side will have the same form as for  $z(t)$ , i.e., it will describe a resonator with coordinate  $\delta z(t)$  and eigenfrequency  $\omega_0$ . In the right-hand-side, however, the term  $-m\gamma z^3$  will lead to the term  $-3m\gamma [z_{\text{vibra}} \cos(\omega t + \phi)]^2 \delta z(t)$ . When averaged over the period  $2\pi/\omega$ , this term leads to the shift of the vibration frequency for  $\delta z(t)$  of the form  $\omega_0 \rightarrow \omega_0 + 3\gamma z_{\text{vibra}}^2 / 4\omega_0$ . This is the well-known frequency shift of a nonlinear oscillator with vibration amplitude. A systematic treatment [41], which does not use the linearization, shows that, if  $z_{\text{vibra}}$  is the amplitude of eigenvibrations, to the lowest order in  $z_{\text{vibra}}^2$  the frequency shift is  $3\gamma z_{\text{vibra}}^2 / 8\omega_0$ , cf. Eq. (20) in Methods.

We now note that the vibrations  $z_{\text{vibra}}(\cos \omega t + \phi)$  are in fact eigenvibrations induced by noise. They have random phase  $\phi$  and also random amplitude. The distribution of this amplitude is of the Boltzmann form,  $\propto \exp(-m\omega_0^2 z_{\text{vibra}}^2 / 2k_{\text{B}}T)$  for weak resonator nonlinearity. The spread of the vibration amplitudes leads to the effective spread of the vibration eigenfrequencies, with typical

width

$$\overline{\delta\omega} = 3\gamma_{\text{eff}}k_{\text{B}}T/4m\omega_0^3; \quad (\text{S14})$$

we have replaced here  $\gamma$  with  $\gamma_{\text{eff}}$  to allow for the renormalization of  $\gamma$  by the quadratic-nonlinearity term, see Eq. (21) in Methods.

Supplementary Fig. S3 shows the resonance line-shape measured at 250 K. From the mechanical bandwidth, measured between the two minima that are flanking the main peak, the apparent quality factor is  $\simeq 50$ . We change the driving force by a factor up to 4 and we do not observe a variation of the bandwidth. Using  $\beta = 4.3 \cdot 10^{24} \text{ m}^{-1}\text{s}^{-2}$  and Eq. (21), we get an apparent quality factor of 67 from Eq. (S14).

The spread of the eigenfrequencies (S14) leads to a broadening of the resonator spectrum. We emphasize that this broadening is not related to the vibration decay, it is a result of the interplay of the resonator nonlinearity and fluctuations. Moreover, since the distribution of the squared vibration amplitude, and thus of the vibration eigenfrequency, is exponential, the spectrum is asymmetric. The overall spectrum in the presence of nonlinearity and fluctuations, on the one hand, and decay, on the other hand, is determined by the ratio of  $\overline{\delta\omega}$  and the decay-induced broadening  $\Gamma$ . It can be obtained in an explicit form for an arbitrary  $\overline{\delta\omega}/\Gamma$  [44]. The frequency spread of the type (S14) can come also from the nonlinear coupling of the considered mode to other modes of the resonator [44]. In the context of carbon nanotubes, this latter mechanism has recently attracted significant attention [22].

Whereas the internal nonlinearity of the resonator leads to the change of the shape of the spectrum with increasing temperature, this is not the case for the nonlinearity associated to the quadratic dependence of the capacitance on the resonator displacement (second term in in Eq. (S8)).

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